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Refractive index measurements of a prism at grazing emergence

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Abstract

A new method is reported to find the refractive index(n) of prism materials by measuring only the angle of incidence (i) when the emergent ray grazes the surface at which refraction takes place .A new general equation is obtained at this grazing emergence situation. Different equations have also been derived for the dependence of n on i for ($30^o - 60^o - 90^o$), ($45^o - 45^o - 90^o$) and ($60^o - 60^o - 60^o$) prisms and accurate values have been found. The limit of applications of those equations are examined and tested.

Keyword: Geometrical optics; Prisms; Refractive index; Grazing emergence.
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1. Introduction

A fundamental optics experiment is the investigation of the dispersion of visible light using a prism spectrometer. The refractive index (n) of the prism material can be calculated for various wavelengths (λ) using the relation:

$$n = \sin[(\alpha + D_{\min})/2]/\sin(\alpha/2).(1) \quad (1)$$

where α is the refractive angle of the prism and $D_{\min}$ is the angle of minimum deviation.
It's also possible to verify that n and λ satisfy the Cauchy relation of the form:

$$n = L + M/\lambda^2 \quad (2)$$

where L and M are constants characteristic of the prism material.

An experiment of this kind is of great help for understanding the meaning of refractive index and its variation with the wavelength in the visible region [1]. On the other hand other experiments ,using HeNe laser and other related optical setup, have also been carried out to find the refractive index of a prism from measurement of the incident angle at which total internal reflection occurs [2].

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To increase the skills of the students in the upper level undergraduate optics course Narasimham and Al-Ani [3] have conducted an experiment to evaluate the refractive index (n) of a material with an equilateral $(60^o - 60^o - 60^o)$ prism by measuring the angle of incidence alone at the time when the emergent ray grazes the surface at which refraction takes place. They found that the refractive index depends on the angle of incidence alone. Earlier, Phelps and Jacobson [4], using the pass off condition suggested an elegant and simple method for the measurement of refractive index of right angle $(45^o - 45^o - 90^o)$ prism.

In solid state Physics many glassy materials are designed in a prism shape in order to measure their refractive indices.Indeed several papers and patents were published over the years related to this topic[5-7].

Furthermore, in advance optics and modern microelectronics experiments the knowledge of accurate values of the wavelength-dependent complex refractive index of thin solid films is important from a fundamental and a technological aspect. The refractive index is necessary for the design and modeling of optical components and optical coatings such as interference filters [8]. The determination of the index of refraction of thin solid films (n_f) may be obtained using the Abeles method .This method is rather simple and accurate and the main apparatus required in this technique is a spectrometer modified to achieve a telescope rotation at a twice the rate of the specimen(thin film) and to be focused on this specimen instead of infinity with a monochromatic source of radiation[9].

This method measures the Brewster angle of the film and depends on the fact that for a polarized light with its electric vector in the plane of incidence, the reflectance of the film(index n_f) deposited on a substrate (index n_s) at an angle $\tan \phi_o = n_f / n_o$ is the same as the reflectance of the bare substrate. The medium of the incidence is air (n_o).Therefore coating of the film over half the substrate is required for the measurement of the refractive index. Further details may be obtained from Al-Ani[9].

This paper reports a new general method for measuring the refractive index of any glass prism by measuring the angle of incidence only at grazing emergence of the ray. On the same principle different accurate equations were obtained for different types of prisms and were tested experimentally. The range of application of those equations and their accuracy were also indicated.

2. Experiment

The method uses an optical spectrometer and a sodium discharge lamp as source to evaluate the refractive index n of a glass prism by measuring the angle of incidence alone when the emergent ray grazes (goes parallel to) the surface at which refraction takes place. The grazing emergence and the path of ray are indicated in figures (1-4) by the arrows (PQRCD) of the ray. The experiments consist of finding the angle of incidence i by a spectrometer for the emergent ray to come off along the surface BRC by looking in the direction DCRB. By fixing the direction of the incident ray, the prism can be rotated such that the emergent ray comes along RCD. The experiments are carried out with different glass prisms namely $(60^o - 60^o - 60^o)$, $(45^o - 45^o - 90^o)$ and $(30^o - 60^o - 90^o)$ (See figures 1-3).

3. Results and Discussion

In this section we are going to derive some equations for different prisms that obtain their values of the refractive indices by measuring the angle of the incident light alone.

3.1 Equilateral Prism.

Figure (1) shows an equilateral glass prism. We have:

$$\because i_1 = \theta_1 + r_1$$
$$\theta_1 = i_1 - r_1$$
$$\because r_1 + \theta_1 + q = 90^\circ$$
$$and \ \because q = 90^\circ - i_1$$
$$\therefore \theta_2 = 90^\circ - r_2$$
$$\because q + \theta_1 + \theta_2 = 120^\circ$$
$$90^\circ - i_1 + i_1 - r_1 + 90^\circ - r_2 = 120^\circ$$
$$\therefore r_1 + r_2 = 60^\circ \qquad \rightarrow (a)$$
$$\because \sin i_1 = n \sin r_1$$
$$\sin i_2 = \sin 90^\circ = 1 = n \sin r_2 \qquad \rightarrow (b)$$
$$\sin r_2 = \tfrac{1}{n}$$
$$r_1 = 60^\circ - r_2$$
$$\therefore \sin r_1 = \sin\left(60^\circ - r_2\right) = \frac{\sqrt{3}}{2}\cos r_2 - \frac{1}{2}\sin r_2 \qquad \rightarrow (c)$$
$$\because \sin^2 r_2 + \cos^2 r_2 = 1$$
$$\cos r_2 = \left(1 - \sin^2 r_2\right)^{\frac{1}{2}}$$
$$\cos r_2 = \left(1 - \frac{1}{n^2}\right)^{\frac{1}{2}} = \frac{\left(n^2 - 1\right)^{\frac{1}{2}}}{n} \qquad \rightarrow (d)$$

subsitiuti ng (d) *in* (c) *we find* :

$$\sin r_1 = \frac{\sqrt{3}}{2}\frac{\left(n^2 - 1\right)^{\frac{1}{2}}}{n} - \frac{1}{2}\frac{1}{n}$$

and

$$\sin r_1 = \frac{\left[3\left(n^2 - 1\right)\right]^{\frac{1}{2}} - 1}{2n}$$

by multiplyin g both sides by μ *and through equation* (b) *we find* :

$$n \sin r_1 = \sin i_1$$

therefore $\sin i_1 = \dfrac{\left[3\left(n^2 - 1\right)\right]^{\frac{1}{2}} - 1}{2}$

By rearrangin g the above we get :

$$n = \left[1 + \frac{1}{3}\left(1 + 2\sin i_1\right)^2\right]^{\frac{1}{2}} \qquad (3)$$

Narasimham and Al – Ani [3] *reported similar equation and found a value of* i_1 *is about* 30° *yielding a value of* $n = 1.5$.

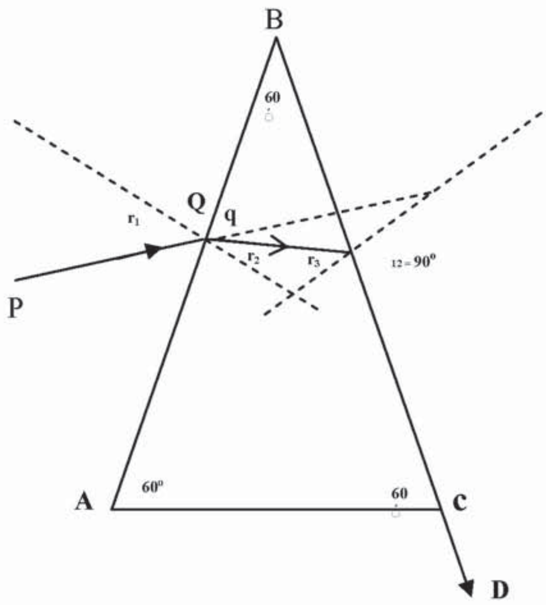

Fig. 1: Incident ray PQ emerges along RCD after refraction though the prism ($60^{o}-60^{o}-60^{o}$).

3.2 Right Angle Prism

Figure (2) shows the measurement of refractive index of a material with a ($45^{o}-45^{o}-90^{o}$) prism .We have:

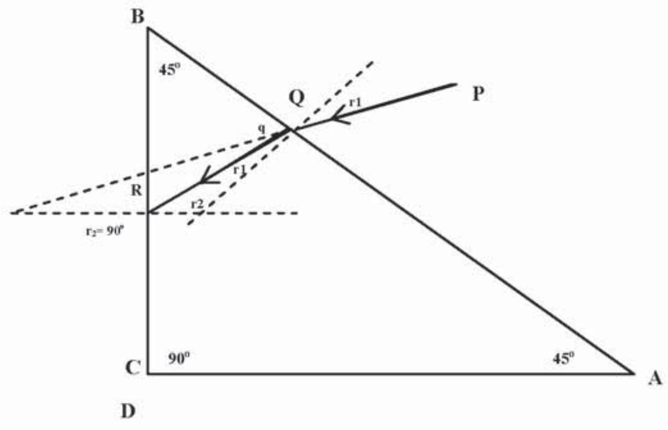

Fig.2 :Incident ray PQ emerges along RCD after refraction though the prism (45°-45°-90°).

$$\because i_1 = \theta_1 + r_1$$

$$\theta_1 = i_1 - r_1 \qquad \rightarrow (a)$$

$$\because q = 90^\circ - \theta_1 - r_1$$

$$= 90^\circ - i_1$$

$$Then \quad 90^\circ - i_1 + \theta_1 + \theta_2 = 135^\circ$$

$$\because \theta_2 = 90^\circ - r_2$$

$$\therefore 90^\circ - i_1 + \theta_1 + 90^\circ - r_2 = 135^\circ$$

using equation (a) above we reach :

$$r_1 + r_2 = 1 \qquad \rightarrow (b)$$

from Snell's law we have :

$$\sin i_1 = n \sin r_1$$

$$\sin i_2 = \sin 90^\circ = 1 = n \sin r_2 \qquad \rightarrow (c)$$

$$\because r_1 = 45^\circ - r_2 \Rightarrow \sin r_1 = \sin\left(45^\circ - r_2\right)$$

$$\sin r_1 = \tfrac{1}{\sqrt{2}} \cos r_2 - \tfrac{1}{\sqrt{2}} \sin r_2 = \tfrac{1}{\sqrt{2}}\left(\cos r_2 - \sin r_2\right) \qquad \rightarrow (d)$$

from equation (c) we have :

$$\sin r_2 = \tfrac{1}{n}$$

and

$$\cos r_2 = \frac{\left(n^2 - 1\right)^{\frac{1}{2}}}{n}$$

$$since \quad \cos r_2 = (1 - \sin^2 r_2)^{\frac{1}{2}} = (1 - \frac{1}{n^2})^{\frac{1}{2}}$$

and by subsituting those results in equation (d) we find :

$$\sin r_1 = \frac{1}{\sqrt{2}}\left[\frac{\left(n^2 - 1\right)^{\frac{1}{2}}}{n} - \frac{1}{n}\right]$$

$$= \frac{1}{\sqrt{2}}\left[\frac{\left(n^2 - 1\right)^{\frac{1}{2}} - 1}{n}\right]$$

By multiplying both sides by n and through equation (c) we find :

$$\sin i_1 = n \sin r_1$$

$$n \sin r_1 = \frac{1}{\sqrt{2}}\left[\left(n^2 - 1\right)^{\frac{1}{2}} - 1\right] = \sin i_1$$

By rearranging the above we obtain :

$$n = \left[1 + \left(1 + \sqrt{2} \sin i_1\right)^2\right]^{\frac{1}{2}} \quad) \qquad (4)$$

$$\because i_1 = \theta_1 + r_1$$
$$\theta_1 = i_1 - r_1$$
$$q = 90^\circ - i_1$$
$$\theta_2 = 90^\circ - r_2$$
$$\therefore q + \theta_1 + \theta_2 = 150^\circ$$
or
$$r_1 + r_2 = 30^\circ \qquad \rightarrow (a)$$
$$\because \sin i_1 = n \sin r_1$$
$$\sin i_2 = \sin 90^\circ = n \sin r_2 \;] \qquad \rightarrow (b)$$
$$\sin r_2 = \tfrac{1}{n} \qquad \rightarrow (c)$$
$$\because \sin^2 r_2 + \cos^2 r_2 = 1$$
$$\cos r_2 = \left(1 - \sin^2 r_2\right)^{\frac{1}{2}} = \frac{(n-1)^{\frac{1}{2}}}{n} \qquad \rightarrow (d)$$
$$\because r_1 = 30^\circ - r_2 \Rightarrow \sin r_1 = \sin\left(30^\circ - r_2\right)$$
$$\sin r_1 = \tfrac{1}{2}\cos r_2 - \tfrac{\sqrt{3}}{2}\sin r_2 \qquad \rightarrow (e)$$
by substituting equations (c) and (d) in (e) we find :
$$\sin r_1 = \frac{1}{2}\frac{\left(n^2-1\right)^{\frac{1}{2}}}{n} - \frac{\sqrt{3}}{2}\frac{1}{n} = \frac{\left(n^2-1\right)^{\frac{1}{2}} - \sqrt{3}}{2n}$$
by multiplying both sides by μ we get :
$$n \sin r_1 = \frac{\left(n^2-1\right)^{\frac{1}{2}} - \sqrt{3}}{2} = \sin i_1$$
$$\sin i_1 = \frac{\left(n^2-1\right)^{\frac{1}{2}} - \sqrt{3}}{2}$$
we have reached to the following equation :
$$n = \left[1 + \left(\sqrt{3} + 2\sin i_1\right)^2\right]^{\frac{1}{2}} \qquad (5)$$

This equation (5) was not reported earlier to the best of our knowledge.

3.4 General Formula

Now we are going to obtain a more general formula using figure (4) and knowing that α is the refracting angle of the prism .We have:

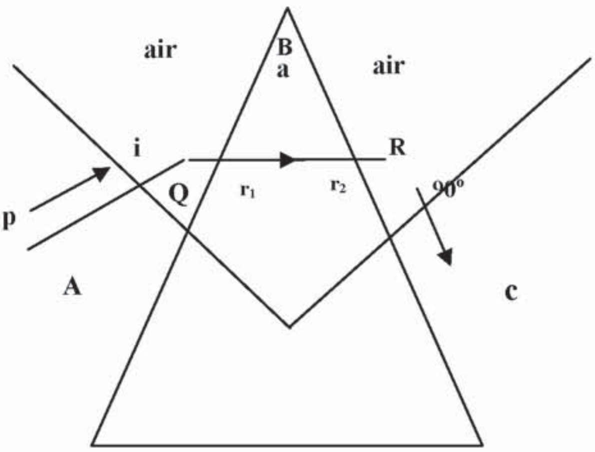

Fig. 4: Incident ray PQ emerges along RCD after refraction though the prism (ABC).

$r_1 + r_2 = \alpha \Rightarrow r_1 = \alpha - r_2$

$\sin r_1 = \sin(\alpha - r_2)$

and

$$\sin r_1 = \sin\alpha \cos r_2 - \cos\alpha \sin r_2 \qquad (6)$$

From Snell's law

$\sin r_2 = \frac{1}{n}$

and $\left(1 - \cos^2 r_2\right)^{1/2} = \frac{1}{n}$

or $\cos r_2 = \frac{\left(n^2 - 1\right)^{1/2}}{n}$

By combining the above equations we obtain:-

$n \sin r_1 = \left(n^2 - 1\right)^{1/2} \sin\alpha - \cos\alpha$

At a point of the incident light entered the prism

$1 \sin i = n \sin r$

Hence

$\sin i = \left(n^2 - 1\right)^{\frac{1}{2}} \sin\alpha - \cos\alpha$

$$n = \left[1 + \left(\frac{\sin i + \cos\alpha}{\sin\alpha}\right)^2\right]^{\frac{1}{2}}$$

$$\therefore n(i,\alpha) = [1 + (\frac{\sin i}{\sin\alpha} + \frac{1}{\tan\alpha})^2]^{1/2} \qquad (7)$$

Equation (7) reported for the first time obtains the refractive index of any prism (knowing its refracting angle α) by measuring the angle of incidence (i) only when the emergent ray grazes the surface at which refraction occurs i.e. on the surface BRCD in figure (4).

When this general equation has applied to different prism's angle: $\alpha = 45^o, 60^o$ and 30^o we have arrived exactly to the same equations obtained above i.e. equations (3), (4), (5) for $(60^o - 60^o - 60^o)$, $(45^o - 45^o - 90^o)$ and $(30^o - 60^o - 90^o)$ prisms respectively.

For example using equation (7) at $\alpha = 45^o$ we have

$$n = [1 + (\frac{\sin i}{1/\sqrt{2}} + \frac{1}{1})^2]^{1/2} = [1 + (\sqrt{2}\sin i + 1)^2]^{1/2}$$

which is equivalent to equation(4) and so on for other prisms (cases).

3.5 Limits and Applications

To examine those equations some calculations were done. Those equations were further examined through those calculations using figure (4):

at $\alpha = 60^o$ and $n = 1.5$ the value of r_2 is:

$$n \sin r_2 = (1)\sin 90^o = 1$$

$$\sin r_2 = \frac{1}{n}$$

$$r_2 = \sin^{-1}(\frac{1}{n}) = \sin^{-1}(\frac{1}{1.5}) = 41.8^o$$

Therefore the value of r_1=18.2°, using the relation at a point of the incident light

$$(1)\sin i = n \sin r_1$$

$$i = \sin^{-1}[n \sin r_1]$$

$$i = 27.92^o$$

This indicates that the prism of $\alpha = 60^o$ and $n = 1.5$, its angle of incident, at which the appeared ray goes parallel to the surface of refraction, is i=27.92°. Hence, when using relation (3) at $\alpha = 60^o$

$$n = \left[1 + 1/3\left(1 + 2\sin 27.92^\circ\right)^2\right]^{1/2}$$

we get n *equal to* 1 . 5 .This is the same value that was obtained earlier. These results prove the reliability of our general equation (7).

Taking $\alpha = 45^o$, the value of $i = 4.79^o$. But for $\alpha = 30^o$ the value of $i = -17.9^o$ which is not applicable. Therefore the last case can be only applied for $n \geq 2$.

The above results indicate that there is a limitation on application of equation (7) on all prisms and only by obeying the following condition:

$$(n^2 - 1)^{1/2} \geq \frac{1}{\tan \alpha} \quad . \tag{8}$$

Or

$$(n^2-1)^{1/2} \geq \cot\alpha \tag{9}$$

In that case, the angle (i) where grazing occurs will be zero i.e the incident ray is perpendicular on the prism surface. So when $\alpha = 30^o$ the refractive index of the glass prism must equal to 2 and $i = 0^o$. Therefore for $n = 1.5$, equation (5) can be applied only on prisms with angles in the range $41.8^o \leq \alpha \prec 90^o$ (using equations 8 or 9). Higher values of n were also studied and their α –ranges were calculated and listed in Table I.

Table I: Prisms with values of n and the ranges of α for application of equation (7)

n	Range of α
1.5	$41.8^o \leq \alpha < 90^o$
1.55	$40.2^o \leq \alpha < 90^o$
1.6	$38.7° \leq \alpha < 90°$
1.9	$31.8^o \leq \alpha < 90^o$

Therefore equation (7) can be applied to a glass prism with : $\alpha = 30^o$ provided its $n \geq 2$; for $\alpha = 45^o$ when its $n \geq 1.414$ and for $\alpha = 50^o$ when its $n \geq 1.31$. But this equation (7) is best applied to the equilateral prism when its refractive index as small as $n \geq 1.16$ .The prism of α=45° is next in preference. It is, thus, concluded that as α is increased, equation (7) can be successfully applied even for prisms with low refractive indices.
The minimum index measurable with a given apex angle is easily derivable from our general result:

$$n(i,\alpha) = \left[1 + \left(\frac{\sin i + \cos\alpha}{\sin\alpha}\right)^2\right]^{\frac{1}{2}}$$

The minimum measurable index occurs when the angle of incidence is zero.In that case the index is:

$$n(0,\alpha) = n_{\min} = \left[1 + \left(\frac{\cos\alpha}{\sin\alpha}\right)^2\right]^{\frac{1}{2}} = (1+\cot^2\alpha)^{\frac{1}{2}} = \csc\alpha$$

Thus our experimental results are consistent with this.

It should be noted that our method is easier to demonstrate and more general than other reported experiments [2,4]. However, it does not need costly equipment to carry out such as other setups. Furthermore, the reported equation in this work is general, simple and elegant and need not elaborated calculations to find the refractive index of the prism under consideration. This method may be extended for other prisms materials and by using a different light source with more emission lines, such as mercury, to determine the constants in the Cauchy relation part of the experiment.

4. Conclusion

This paper presents an experiment for the measurement of refractive index of any prism by finding the angle of incidence only by a spectrometer for the emergent ray to come off along the surface of refraction. A general equation was obtained for that purpose and successfully applied to different types of prisms materials. Our results is in line with earlier findings.

The present method is simpler than other reported experiments, easy to demonstrate and has not been reported earlier to the best of our knowledge.

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